Performance of Optical Networks With Limited Reconfigurability

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***Abstract*—We investigate the blocking performance of all-optical reconfigurable networks with constraints on reconfiguration brought by reconfigurable optical add/drop multiplexers (ROADMs) and tunable transponders. Considering a fully reconfigurable ROADM and limited tunable transponders at each ROADM port, we develop an analytical model to calculate call blocking probability in a network of arbitrary topology for two different models for transponder sharing within a node: share-per-link (SPL) and share-per-node (SPN). In such a configuration, limited tunable transponders determine the set of wavelengths that can be added/dropped at a reconfigurable node. A lightpath can only be established if a transponder on both ends can tune to the same available wavelength along the route. We call this *wavelength termination constraint*. The number of transponders (as many as ports) and the waveband size (the range of wavelengths over which a transponder is tunable) are the key parameters of the model, assuming that wavebands are randomly assigned to transponders. We also present a heuristic algorithm for assigning wavebands to transponders at each node and modify the analytical model to approximate the performance of the algorithm. We present simulation results to validate our model and also show results for share-per-link and share-per-node models with various sets of parameters. We show that limited tunable transponders give the same performance with widely tunable transponders in terms of blocking. We also show that transponder waveband assignment to limited tunable transponders is an important factor determining the blocking.**

***Index Terms*—Limited tunable transponders, reconfigurable optical networks, reconfigurable optical add/drop multiplexer (ROADM).**

I. Introduction

OPTICAL networks today are based on wavelength division multiplexing (WDM) technology that offers high capacity to carry information traffic. In WDM networks, lightpaths are established between the stations that need to communicate. One station transmits the information on a certain wavelength, and the destination station receives on that wavelength. This functionality is achieved through optical components called optical add/drop multiplexers (OADMs), which can add/drop certain wavelengths at nodes and bypass the other wavelengths. With OADMs, optical–electrical–optical (O–E–O) conversion is not necessary [1]. Fixed OADMs (FOADMs) are the first solution developed, but they can be used to establish permanent lightpaths only since they are set to add/drop fixed wavelengths. FOADMs are not suitable in the case of dynamically changing network traffic. Manually configuring the nodes with FOADMs in order to change the lightpaths in the network is a burden on network operators. This results in high operational expenditures (OpEx) while the cost of the FOADMs (CapEx) is relatively low.

In order to dynamically adapt to changes in traffic demands, carriers are considering reconfigurable optical networks that allow remote configuration of lightpaths. Such reconfigurability in a network does not require any preplanning for the carriers, and lightpaths can be established or torn down within a matter of seconds whenever wanted. The key technology enabling reconfigurability in an optical network is the reconfigurable optical add/drop multiplexer (ROADM). Different from FOADMs, ROADMs do not permanently add/drop the same wavelengths, but they can be remotely configured to add/drop different wavelengths at different times. Allowing the bypass traffic to pass through without O–E–O conversion reduces the overall equipment costs and also loosens the capacity bottleneck caused by electrical signal processing [2]. ROADMs enable carriers to offer a flexible service and provide significant savings in OpEx and CapEx [3]. There are several papers in the literature explaining various ROADM architectures and technologies [1], [4]. We review the three main ROADM architectures and corresponding technologies in Section II. ROADM types may differ in wavelength granularity and channel bandwidth. Another important factor is whether they have wavelength-specific (colored) or colorless add/drop ports [4]. Different types also have different connectivity capabilities. Connectivity specifies how many optical links a ROADM can support. Accordingly, ROADMs can be classified as degree-2 ROADMs or multiple-degree ROADMs. Degree-2 ROADMs are most commonly used in ring networks, while higher degree ROADMs can be used to physically connect mesh networks. In [3], it is shown that multiple-degree ROADMs enabling interconnection of several ring networks can reduce network cost (CapEx) significantly. A similar categorization of ROADMs is done in [2], and they are compared according to their wavelength routing capabilities. ROADM applications in metro and regional networks, and some future network architectures depending on ROADMs are discussed in [5]. In order to achieve full reconfigurability, ROADMs are used in conjunction with tunable transceivers (transponders). Tunable transceivers are required to transmit/receive the added/dropped wavelengths at each ROADM port. They utilize tunable filters

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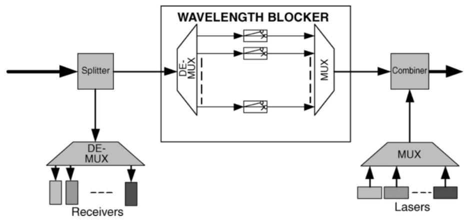

Fig. 1. A ROADM using the broadcast-and-select architecture.

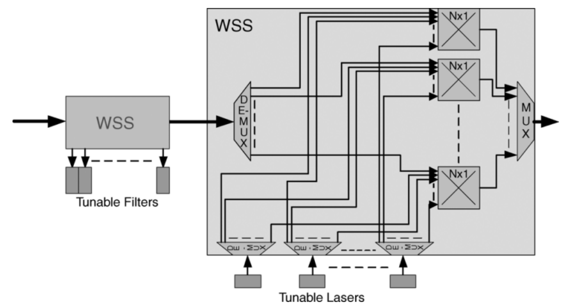

Fig. 2. A ROADM using the wavelength selective switch architecture.

and lasers to terminate the drop wavelengths and to launch the add wavelength signals.[1]

In this paper, we consider several ROADM architectures and transponder types, and we develop an analytical model for calculating blocking depending on the level of reconfigurability. We quantitatively illustrate the benefits of reconfigurability considering different levels of reconfiguration in an optical network. We consider a dynamic traffic model where lightpath establishment and teardown requests arrive according to a stochastic process. If an arriving lightpath request cannot be established by the reconfigurable network (without changing the already established lightpaths), then it is blocked. The blocking metric used together with such a stochastic model is widely used in the literature in the context of optical networks.

The rest of the paper is organized as follows. In Section II, we review ROADM architectures and technologies and also transponder types and capabilities. We also review related work in the literature on calculating the blocking performance in optical networks. Section III describes the network and traffic model. In Section IV, we present an analytical model for two different transponder sharing models and also for a heuristic developed for transponder waveband assignment. The model is validated by comparing with simulation results in Section V, where we also present our performance evaluation results, and the paper is concluded in Section VI.

II. Background

A. *ROADMs and Tunable Transponders*

ROADMs have the capability to be remotely reconfigured to add or drop different wavelengths at different times. Such flexibility frees the network operator from extensive preplanning and allows the offering of flexible services to customers, such as *lambda on demand*. ROADMs are built on three different architectures: broadcast-and-select, demux-switch-mux, and wavelength selective switch (WSS).

In Fig. 1, a ROADM using the broadcast-and-select architecture is shown. This architecture consists of a drop module, an add module, and a wavelength blocker (WB) in between. In the drop module, the incoming fiber is divided into two equivalent signals by a coupler, with one of them going to the drop port and the other toward the express path. WB blocks the desired wavelength that is to be dropped, and a wavelength is added on the express path from the add ports by using another coupler [4]. A demultiplexer/multiplexer is used in the drop/add port. With this configuration, only a certain wavelength can be added/dropped at one port (colored ports). In order to have colorless ports, the demultiplexer and the multiplexer need to be replaced with a splitter and a combiner, respectively [1], [4]. This architecture has a lower cost, but the insertion loss increases with the number of wavelengths dropped [4]. WB is built based on liquid crystal (LC) or MEMS technologies, but LC is the lower cost solution. These ROADMs are degree-2, and they cannot be upgraded to support multiple degrees.

ROADMs using the demux-switch-mux architecture [1] use arrayed waveguide gratings (AWGs) for multiplexing and demultiplexing together with switches and attenuators. Add/drop switching follows after demultiplexing the incoming signal. This architecture is built on planar lightwave circuit (PLC) technology, and its components are integrated on a chip having a smaller size. ROADMs in this configuration can drop each wavelength to their specific (colored) ports, and they are not degree-upgradeable [1].

ROADM using the WSS architecture is the most attractive solution today due to its full reconfigurability (i.e., add/drop any wavelength from/to any port; colorless ports). ROADMs using this architecture are mostly built on MEMS technology, but with recent advances, WSS architectures have been built with LC technology as well. A MEMS spatial switching element is used to switch any wavelength to any of the ports, as shown in Fig. 2. The first WSS module handles switching to the drop ports, while the second WSS module switches wavelengths from the add ports to the output fiber. Optical branching is easier in WSS ROADMs; it can support up to degree 8. A hybrid architecture consisting of one WSS module with colorless drop ports and colored add ports or vice versa is proposed in [6].

In terms of wavelength granularity, WB ROADMs may have a channel spacing as low as 25 GHz, whereas PLC-based ROADMs have 100 GHz, and WSS ROADMs have 50 to 100 GHz channel spacing.

Another factor determining reconfigurability is the transponders used at the add/drop ports. We can classify the transponders into three categories: 1) fixed transponders; 2) limited or narrowly tunable transponders; and 3) widely tunable transponders. Fixed transponders transmit or receive at fixed wavelengths that cannot be changed dynamically, whereas tunable transponders can be tuned to different wavelengths. Widely tunable transponders have the ability to tune to the entire range of C-band wavelengths ($\sim$1530 to $\sim$1565 nm) and sometimes even to the L-band ($\sim$1570 to $\sim$1610 nm), whereas limited or narrowly tunable transponders can only tune to a certain wavelength range within the entire wavelength spectrum. The

[1]Transponders incorporate additional electrical/demultiplexing functions over transceivers; we use the term transponders to represent both in this paper.

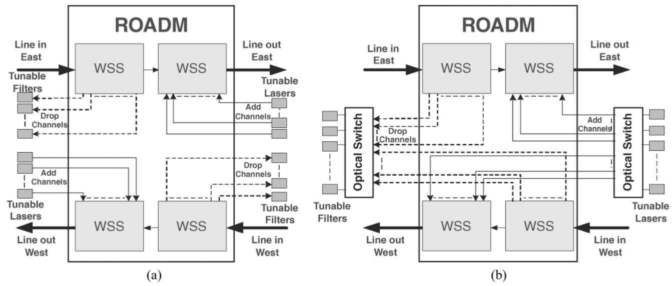

Fig. 3. (a) SPL and (b) SPN transponder models.

number of wavelengths a narrowly tunable transponder can tune to can vary—e.g., four or eight wavelengths in a 32-wavelength system. Transponders are expensive components [7], and carriers are likely to employ a pay-as-you-grow strategy in deploying them. This means that initial transponder deployments may be small in number and limited in tunability. In order to achieve full reconfigurability, the transponders at the colorless ports of a ROADM should be tunable. Tunable transponders can reduce the inventory costs significantly since one transponder can be used for more than one wavelength. Moreover, limited tunable transponders have a lower cost than widely tunable transponders.

There are two models considering the use of transponders within a reconfigurable node. In the first model, there are dedicated transponders for each link's corresponding add/drop ports. This model is called share-per-link (SPL), and it is illustrated in Fig. 3(a). On the other hand, share-per-node (SPN) model allows sharing of transponders within a node among all of the links connected to it. Every add/drop port of any outgoing/incoming link is connected to an optical switch, as seen in Fig. 3(b), and the optical switch can connect any of these ports to any of the transponders.

B. Motivation, Goals, and Related Work

There is very little previous work in the literature analyzing the performance of reconfigurable optical networks considering ROADMs and tunable transponders. In [8], the authors quantified the relationship between the number of fully reconfigurable OADMs that can be replaced by limited ROADMs (that can add/drop wavelengths only over a limited range) and the tuning range for static all-to-all traffic. In [9], the authors consider the tuning process of ROADMs with the constraint that it does not interfere with working wavelengths and provides heuristics to avoid such interference. In [10], blocking performance of WDM networks with limited tunable transponders is investigated. It is shown using simulation results that a network with limited number of transponders with limited tunability at each node can perform close to the case with full number of widely tunable transponders. The same authors developed an analytical model in [11] considering widely tunable transponders at the add/drop ports and showed that a limited number of add/drop ports is sufficient. In [12], for fully reconfigurable networks with limited ports and nonreconfigurable networks, an analytical model is developed for blocking only for ring topology.

In this paper, we develop an analytical model and conduct simulations to obtain the blocking performance. We consider a fully reconfigurable ROADM (i.e., WSS ROADM) with limited tunable transponders at the colorless ports. ROADMs with colored ports can also be adapted into our analytical model by considering only fixed transponders at the add/drop ports.[2] Therefore, the actual constraint on wavelengths that can be received/transmitted by a node comes from the limited tunable transponders. At each port, there is a limited tunable transponder that can add/drop only a certain set of wavelengths. These wavelengths actually determine if a lightpath can be originated or terminated at that node. We call the constraint brought by tunable transponders on the add/drop capability of a reconfigurable node *wavelength termination constraint*. Key parameters directly affecting blocking are the number of ports that a ROADM has and the wavelength range over which narrowly tunable transponders are tunable. Our analytical model is based on these parameters as well as the number of wavelengths in the system and traffic loads on each route.

On the other hand, wavelength blocking performance has been studied widely in the literature (e.g., [13]–[18]). All of these papers consider *wavelength continuity constraint*, which requires a lightpath to be routed using the same wavelength end-to-end when wavelength converters are not present.

III. Network and Traffic Model

There are N nodes in the network that are numbered $1, 2, \ldots, N$ and L bidirectional links interconnecting them (i.e., each link has two fibers in opposite directions). Each fiber carries W wavelengths. D_n denotes the number of links that a node is connected to or the degree of node n, $1 \leq n \leq N$. For any node, we denote the number of transponders per degree by T. Therefore, there are T transponders dedicated for each link connected to a node in the SPL model. There are a total of $T_n = D_n \cdot T$ transponders shared within a node n in the SPN model. By this way, the total number of transponders at a node is equal in both SPL and SPN, and this enables us to make a fair comparison while presenting numerical results. We have $T \leq W$ for both models since there is no advantage offered by having more transponders than wavelengths. Transponders at each port are limited tunable transponders. Each transponder is tunable to a contiguous set of wavelengths (waveband) of the same size. We call the size of this set as *tuning range*, and its value is denoted by Θ. The extreme cases are obtained with $\Theta = 1$ and $\Theta = W$ corresponding to fixed transponders and widely tunable transponders, respectively. The number of nonoverlapping wavebands in our system is $K = W/\Theta$. We assume K to be an integer.

Each node is equipped with a fully reconfigurable ROADM (i.e., WSS ROADM) that can drop any of the W wavelengths to any of the ports. We have as many transponders in a node as the number of ports.

Fixed routes are calculated between every node-pair according to minimum hop routing. The set of routes is denoted by $\mathcal{R}$. Lightpath requests arrive according to a Poisson process at each route. We assume the same call arrival rate ρ on all routes. This is just for simplicity, and any traffic pattern can be used in our model. Without loss of generality, call durations are

[2]FOADMs and limited ROADMs can also be modeled by using appropriate limited tunable transponders.

exponentially distributed with unit mean. Thus, the Erlang load on each route is ρ.

A. Lightpath Establishment

When a lightpath request arrives, the set of wavelengths which are available end-to-end and to that a transponder on both ends can tune is found. A wavelength is chosen randomly from this set, and the call is established on that wavelength. In this random selection scheme, each wavelength in the set is weighted by the number of available transponders tunable to it at the source node. Other selection strategies can also be applied, but our goal is to understand the impact of reconfigurability rather than to examine differences due to wavelength selection policies. Weighting is applied for performance reasons and also to simplify the analytical model. If the wavelength set is empty, the call is blocked and lost.

B. Transponder Waveband Assignment

In this section, we explain how we choose the wavebands of limited tunable transponders at each node. We adopt a random strategy for this purpose, by which we want to have as many different wavebands as possible at a node. For the SPL model, we do the assignment for every T transponders for each link a node is connected to. There are $\lfloor T/K \rfloor$ transponders assigned to each waveband, and the remaining $b = T \bmod K$ transponders are assigned wavebands randomly out of K wavebands without assigning the same wavebands twice. In particular, for the case of fixed transponders, the wavelengths of the T transponders are randomly selected out of the W wavelengths. For $T \leq K$, this scheme becomes completely random assignment.

For the SPN model, the total transponders (T_n) of a node n are chosen in the same fashion explained above. Our analytical model is developed based on this random assignment strategy. However, we examine another assignment strategy in Section IV-J and develop an approximate analytical model for that strategy as well.

IV. Analytical Model

We derive the blocking probability for a route $R \in \mathcal{R}$ initially for the SPL model. The network blocking is computed as the average of the blocking probabilities over all routes. In Section IV-G, we develop a similar analytical model for the SPN model. We start with some assumptions and definitions.

A. Assumptions and Definitions

Let $R = (l_1, l_2, \ldots, l_H)$, where l_1 is the first link on R, l_2 the second link, etc. Let s and d be the source and destination of route R. Let (s, l_1) denote the source node and the first link of route R, respectively. Throughout this section, random variables are represented using uppercase letters, and their sample values are represented by the same letter in lowercase. We categorize the wavelengths in l_1 into the following:

1) *Free wavelength*: A wavelength that is not occupied on l_1. We denote the number of such wavelengths as W_f.
2) *Transmittable wavelength*: A free wavelength on l_1 to which an available transponder on s is tunable. The number of transmittable wavelengths is W_t.

The calls using l_1 are also categorized:

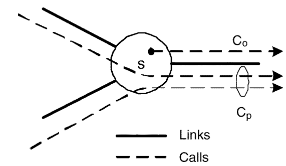

Fig. 4. Illustration of *originating calls* and *passing calls*.

1) *Originating calls*: Calls starting at s and going onto l_1. Let C_o be the number of *originating calls*.
2) *Passing calls*: Calls going onto l_1, but not originating at s. Let C_p be the number of *passing calls*.

The two kinds of calls are illustrated in Fig. 4. In the SPL model, an originating call on l_1 occupies one of the transponders dedicated for the pair (s, l_1). Each of the passing calls and originating calls uses one of the wavelengths in l_1.

Our derivation of blocking probability for R consists of three parts. We first derive the joint distribution of W_t and W_f. The second part utilizes the path model developed in [15] incorporating link-load correlation effects. Let $(l_1, l_2, \ldots, l_j) : j \leq H$ be a route segment of R, and redefine the transmittable wavelengths for the segment as the wavelengths available on all links $l_1, l_2, \ldots, l_j$ and to which an available transponder on (s, l_1) can tune. Similarly, redefine free wavelengths as the wavelengths that are not occupied on the last link l_j of the route segment. Then, the path model in [15] can be used to compute the joint distribution of the transmittable and free wavelengths for $j = 2, 3, \ldots, H$, in an iterative manner. In the third part, we find the blocking probability by computing the probability that none of the available transponders on (d, l_H) can tune to any of the transmittable wavelengths. We explain the details below.

B. The Distribution of W_t

We find the probability that there are w_t transmittable wavelengths on (s, l_1) given that there are c_o and c_p originating calls and passing calls, respectively. Call this probability $P_1(w_t | c_o, c_p)$, and define $a = \lfloor T/K \rfloor$. We know that $b = T \bmod K$ wavebands have $a + 1$ transponders and the rest have a transponders. We randomly distribute c_o originating calls onto transponders first. Note that a waveband will be unavailable for transmission if all of its transponders are occupied. We need to take into account the number of unavailable wavebands as a result of this distribution. Let α_1 and α_2 be the number of wavebands whose $a + 1$ and a transponders, respectively, are fully occupied by some of the c_o calls. We find the number of ways of having exactly α_1 and α_2 unavailable wavebands, which is denoted by $N_2(\alpha_1, \alpha_2 | c_o)$. α_1 and α_2 unavailable wavebands are randomly assigned out of b and $K - b$ wavebands, respectively. Moreover, none of the remaining wavebands should be fully occupied by the remaining originating calls. Distributing the remaining $c_o'' = c_o - \alpha_1(a+1) - \alpha_2 a$ calls to the remaining wavebands in such a way that constitutes a balls-and-urns problem. Consider a set of urns that are arranged as s_1 columns of $a + 1$ urns each and s_2 columns of a urns each, for a total of $\eta = (a+1)s_1 + a s_2$ urns. Let $\zeta(r, s_1, s_2, a)$ be the number of ways of not filling any column of urns fully when r balls are thrown at random into the urns (no more than one ball per urn). Then, $\zeta(.)$ is given by (1) at the bottom of the page, where $\mathrm{up}_i = \min(s_1, \lfloor r/(a+1) \rfloor)$, $\mathrm{up}_j = \min(s_2, \lfloor (r - i(a+1))/a \rfloor)$, and low_j is 1 if i is 0, and 0 otherwise.

Using (1), we calculate

$$N_2(\alpha_1, \alpha_2 | c_o) = \binom{b}{\alpha_1}\binom{K-b}{\alpha_2}\zeta\left(c_o'', \alpha_1', \alpha_2', a\right) \quad (2)$$

where $\alpha_1' = b - \alpha_1$, and $\alpha_2' = K - b - \alpha_2$.

After the assignment of c_o calls (each also uses a wavelength), c_p calls will be distributed randomly to any of the remaining available wavelengths. Note that some wavelengths in a waveband can still be assigned to a passing call, even if that waveband is unavailable for transmission. Passing calls do not require a transponder, but they just use a wavelength. Since we have $T_s \leq W$ for SPL, it can be easily shown that $a + 1 \leq \Theta$, which means α_1 wavebands will still have $\Theta - (a+1)$ wavelengths that can be used by a passing call. Similarly, α_2 wavebands have $\Theta - a$ remaining wavelengths for passing calls. Let us denote the passing calls in unavailable and available wavebands by c_p' and $c_p'' = c_p - c_p'$, respectively. Assuming random wavelength assignment, the number of ways this occurs is given by

$$N_3\left(c_p' | \alpha_1, \alpha_2\right) = \binom{\alpha_1\left(\Theta - (a+1)\right) + \alpha_2(\Theta - a)}{c_p'} \times \binom{(K - \alpha_1 - \alpha_2)\Theta - c_o''}{c_p''}. \quad (3)$$

The number of transmittable wavelengths can be calculated using the values of $\alpha_1, \alpha_2, c_p'', c_o''$ as $w_t = (K - \alpha_1 - \alpha_2)\Theta - c_p'' - c_o''$. We illustrate this calculation with an example in Fig. 5 with the parameters $T = 10$, $\Theta = 4$, $W = 16$. We have $K = W/\Theta = 4$, $a = \lfloor T/K \rfloor = 2$, and $b = T \bmod K = 2$. Therefore, there are $a+1 = 3$ transponders for $b = 2$ wavebands and $2(= a)$ transponders for the other $2(= K - b)$ wavebands. The wavebands are shown as the large rectangles and wavelengths as small rectangles. In the first waveband, three originating calls occupy three transponders and wavelengths. There is one available wavelength, yet it is not transmittable since all three transponders in that waveband are occupied. The second and third wavebands are available for transmission since they have an available transponder. One passing call and one originating call occupy two wavelengths in the second waveband, so it has two transmittable wavelengths. In the third waveband, only one wavelength is transmittable. The last waveband is also unavailable for transmission. Therefore, we have $w_t = 3$ transmittable wavelengths in this example. Note that if the passing call in the last waveband was in the second or third waveband instead, then the number of transmittable wavelengths would be one fewer.

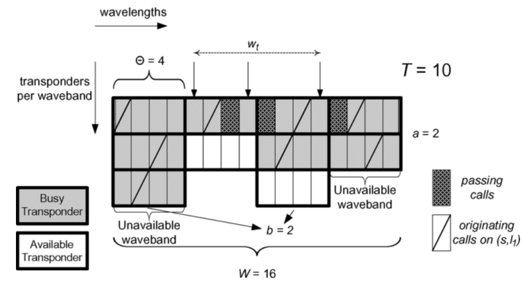

Fig. 5. An example to illustrate how transmittable wavelengths are determined.

The final expression for the total number of ways of having w_t is calculated as follows:

$$\begin{aligned} N_4(w_t | c_o, c_p) &= \sum_{\left\{\alpha_1, \alpha_2, c_p' : (K - \alpha_1 - \alpha_2)\Theta - c_p'' - c_o'' = w_t\right\}} N_2(\alpha_1, \alpha_2 | c_o) \\ &\quad \times N_3\left(c_p' | \alpha_1, \alpha_2\right), \end{aligned} \quad (4)$$

for $w_t = 0, 1, \ldots, \min(T\Theta, W)$.

Continuing the analysis, we now consider the overall number of ways of assigning c_o and c_p calls. Each of the c_o and c_p calls is on a different wavelength, and each of the c_o calls takes up a transponder on (s, l_1). Because of random wavelength/band assignment to transponders and random wavelength assignment to calls, the total number of distinct ways in which transponders are assigned to the c_o calls and the $W - c_o$ wavelengths to the c_p calls is

$$N_1(c_o, c_p) = \binom{K}{b}\binom{T}{c_o}\binom{W - c_o}{c_p}. \quad (5)$$

The probability that there are w_t transmittable wavelengths given c_o originating and c_p passing calls is given by

$$P_1(w_t | c_o, c_p) = \frac{\binom{K}{b} N_4(w_t | c_o, c_p)}{N_1(c_o, c_p)} = \frac{N_4(w_t | c_o, c_p)}{\binom{T}{c_o}\binom{W - c_o}{c_p}}. \quad (6)$$

The multiplication by $\binom{K}{b}$ in the numerator of (6) is to account for the random choice of the wavebands that have an extra transponder.

Special Cases:

1) When $\Theta = 1$, we have $\alpha_1 = c_o$ and $\alpha_2 = W - T$, and $\zeta(.) = 1$.

$$\zeta(r, s_1, s_2, a) = \begin{cases} 0, & \text{if } r > as_1 + (a-1)s_2 \\ \binom{\eta}{r}, & \text{if } r < a \\ \binom{\eta}{r} - \sum\limits_{i=0}^{\mathrm{up}_i} \sum\limits_{j=\mathrm{low}_j}^{\mathrm{up}_j} \binom{s_1}{i}\binom{s_2}{j} \cdot \zeta\left(r - (a+1)i - aj, s_1 - i, s_2 - j, a\right), & \text{otherwise} \end{cases} \quad (1)$$

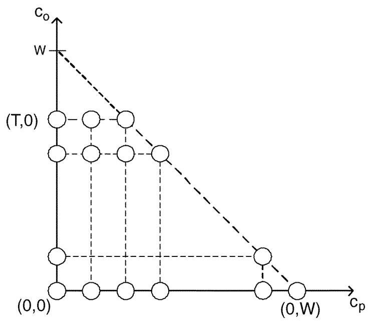

Fig. 6. State space for the 2-D Markov chain of C_o and C_p.

Using these values, (6) simplifies to

$$P_1(w_t|c_o,c_p) = \frac{\binom{W-T}{w_t+c_o+c_p-T}\binom{T-c_o}{w_t}}{\binom{W-c_o}{c_p}}. \quad (7)$$

This can be shown to be the same as the expression for fixed transponders in [19].

2) When $\Theta = W$ as in widely tunable transponders, (6) reduces to: For $c_o = T$, $P_1(w_t = 0|c_o,c_p) = 1$, and 0 otherwise; for $c_o < T$, $P_1(w_t = W - c_o - c_p|c_o,c_p) = 1$, and 0 otherwise.

C. The Joint Distribution of C_o and C_p

In this section, we calculate the joint distribution of the passing calls and the originating calls on the first link l_1. Due to the assumption that calls arrive at each route according to a Poisson process, we can model passing calls and originating calls as a two-dimensional (2-D) Markov chain. The state space for this model is shown in Fig. 6. ρ_o and ρ_p are the Erlang loads of originating and passing calls, respectively, on l_1 of route R. These can be easily obtained because the routing is fixed and the loads on routes are given.

This distribution is calculated as follows:

$$P_2(c_o,c_p) = \frac{\left(\rho_o^{c_o}\rho_p^{c_p}\right)/(c_o!c_p!)}{\Delta} \quad (8)$$

where the normalization factor Δ is obtained by

$$\Delta = \sum_{c_p=0}^{\min\left(W,T_p^{\max}\right)} \sum_{c_o=0}^{\min(W-c_p,T)} \left(\rho_o^{c_o}\rho_p^{c_p}\right)/(c_o!c_p!). \quad (9)$$

$T_p^{\max}$ in (9) is an upper limit on the number of passing calls. This upper limit is determined by the total number of transmitters of distinct node–link pairs (s', l_1') from which a route $R' \in \mathcal{R}$ originates, and R' passes through link l_1 in SPL. If there are e such pairs, then $T_p^{\max} = eT$.

D. The Joint Distribution of W_t and W_f

Using P_1 and P_2, the joint distribution of W_t and W_f can be calculated by the following equation:

$$P_3(w_t,w_f) = \sum_{c_o=0}^{T} P_1(w_t|c_o,c_p)P_2(c_o,c_p) \quad (10)$$

where $c_p = W - c_o - w_f$.

E. Extension to Last Link of Route

Using $P_3(w_t,w_f)$ from the previous section, our goal is to obtain the the joint probability of transmittable wavelengths over the entire route R and free wavelengths on the last link l_H. Calculating the distribution of the number of transmittable wavelengths and free wavelengths for every route segment (denoted by $P_3^{(i)}(w_t,w_f)$) iteratively, we finally obtain $P_3^{(H)}(w_t,w_f)$ on l_H. The iterative method in [15] assumes the wavelength usages on adjacent links are correlated, and the correlation is captured by using the two-link model. This model considers the adjacent link pair (l_i, l_{i+1}) on R and splits the calls using one or both of the links into three parts: 1) C_l is the number of leaving calls, i.e., calls that use l_i and leave the route R at the intermediate node; 2) C_c is the number of continuing calls from l_i to l_{i+1}; and 3) C_n is the number of joining calls, i.e., calls that join route R at the intermediate node and use only l_{i+1}. The Erlang loads for these types of calls can also be readily obtained. Then, the joint distribution of (C_l, C_c, C_n) is obtained in [15] using a three-dimensional (3-D) Markov chain approximation. Call this distribution $P_4(C_l, C_c, C_n)$. The additional constraint on (C_l, C_c, C_n) is that their numbers are not only less than or equal to W, but also they are limited by the number of transponders at the source nodes of the routes contributing to such calls. Given $P_3^{(i)}(w_t,w_f)$, [15] uses $P_4(.)$ to obtain $P_3^{(i+1)}(w_t,w_f)$.

F. End-to-End Blocking Probability

Our end-to-end blocking probability calculation is based on the number of free receivers of the (d, l_H) pair, denoted by R_f. Before we find the distribution of R_f, we define *terminating calls* as the calls that go through the link l_H and terminate at the node d. Let C_t be the number of terminating calls and ρ_t be the load of such calls. The definition of the passing calls (C_p) is slightly modified to be the calls passing through l_H but not terminating at the node d and ρ_p is the load. Now, we can calculate $P_2(c_t,c_p)$ with those load values and obtain the distribution of R_f conditioned on W_f on (d, l_H) with the following equation:

$$P_5(r_f|w_f) = \frac{P_2(T-r_f, W-T+r_f-w_f)}{\sum_{c_p=\max(W-T-w_f,0)}^{W-w_f} P_2(W-c_p-w_f, c_p)}. \quad (11)$$

In (11), since we have r_f free transponders, then we should have $c_t = T - r_f$ terminating calls, and the value of c_p is calculated as $W - w_f - c_t$.

The probability that there is at least one transmittable wavelength end-to-end can be calculated using the distribution in (7) with the given values of r_f and w_t as follows:

$$P_6(.|w_t,r_f) = 1 - P_1(0|T-r_f, W-T+r_f-w_t). \quad (12)$$

It involves finding the probability that there is no receivable wavelength (defined similar to transmittable wavelength) and then taking the complement. In this calculation, we include $w_f - w_t$ free wavelengths in the total value of c_p since they also reduce the number of wavelengths on the last link, just like passing calls. Then, the nonblocking probability for route R is given by

$$B_R' = \sum_{w_f=1}^{W}\sum_{w_t=1}^{w_f}\sum_{r_f=1}^{T} P_3^{(H)}(w_t,w_f)P_5(r_f|w_f)P_6(.|w_t,r_f)$$

and the blocking probability is $B_R = 1 - B_R'$.

G. Blocking Analysis for the SPN Model

In this section, we apply the above analysis for the share-per-node model. We first define *total originating calls* as all the calls starting at s and going onto any of the links connected to s. Note that *originating calls* is a subset of the *total originating calls*. C_s denotes the number of *total originating calls* from s. The difference in analysis for SPN is that each of the *total originating calls* occupies one of the transponders that is in the common pool of transponders of s. There are $T_s = TD_s$ transponders at node s. Therefore, the analysis for the number of ways of having exactly α_1, α_2 unavailable wavebands now depends on the value of c_s. We have $N_2(\alpha_1, \alpha_2|c_s)$ in which $a = \lfloor T_s/K \rfloor$ and $b = T_s \bmod K$.

At this point, we now consider the assignment of c_o calls out of c_s calls. We distinguish between the c_o calls in the unavailable and available wavebands as c'_o and $c''_o = c_o - c'_o$, respectively. The number of ways of having c'_o is given by

$$N_5\left(c'_o|\alpha_1, \alpha_2\right) = \binom{\alpha_1(a+1)+\alpha_2 a}{c'_o}\binom{c_s-\alpha_1(a+1)-\alpha_2 a}{c''_o}. \tag{13}$$

Note that (13) is valid only for the case $T_s \leq W$. If we consider the wavebands having more than Θ total originating calls, then we can only assign at most Θ originating calls to those wavebands. This can only occur for the case $\lceil T_s/K \rceil > \Theta$, which corresponds to $T_s > W$. However, we approximate this case with (13).

For the assignment of c_p calls, we have

$$N_3\left(c'_p|\alpha_1, \alpha_2, c'_o\right) = \binom{(\alpha_1+\alpha_2)\Theta - c'_o}{c'_p} \times \binom{(K-\alpha_1-\alpha_2)\Theta - c''_o}{c''_p}. \tag{14}$$

Then, the number of ways of having w_t transmittable wavelengths is given by

$$N_4(w_t|c_o, c_p, c_s) = \sum_{\left\{\alpha_1,\alpha_2,c'_o,c'_p:(K-\alpha_1-\alpha_2)\Theta - c''_p - c''_o = w_t\right\}} N_2(\alpha_1, \alpha_2|c_s) \times N_5\left(c'_o|\alpha_1, \alpha_2\right) N_3\left(c'_p|\alpha_1, \alpha_2, c'_o\right) \tag{15}$$

for $w_t = 0, 1, \ldots, \min(T_s\Theta, W)$.

We illustrate this calculation again with a similar example in Fig. 7. We have a degree-2 node with $T = 5$, and the total number of transponders is $T_s = 10$. All the parameters are the same as in the previous example for SPL model. We have $c_o = 4$, $c_p = 4$ and $c_s = 8$. c_o calls are distributed among the c_s calls. We assigned some of the c_s calls on the same wavelength since they could be on different links (we can at most make double assignment since the degree of the node is 2). The first waveband is unavailable, since all three transponders are occupied by some of the c_s calls. The last waveband also has all two of its transponders occupied. The second waveband is available for transmission since only one of its transponders is occupied. There is one passing call on this waveband. Note that the total originating call does not use a wavelength of the second waveband on this link. Therefore, the number of transmittable wavelengths is three. The third waveband has one if its transponders free, so it is also available. One passing call and one originating call occupy wavelengths so the number of transmittable wavelengths is two. We have a total of five transmittable wavelengths in this example.

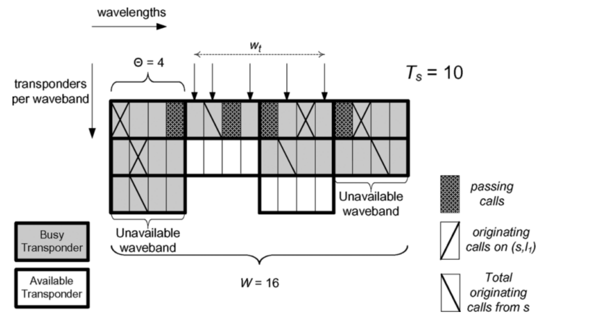

Fig. 7. An example to illustrate how transmittable wavelengths are determined.

Going back to the analysis, the total number of distinct ways of assigning c_s, c_o and c_p calls (again omitting the restrictions on the assignment of c_o calls) is given by

$$N_1(c_o, c_p, c_s) = \binom{K}{b}\binom{T_s}{c_s}\binom{c_s}{c_o}\binom{W-c_o}{c_p}. \tag{16}$$

The probability that there are w_t transmittable wavelengths given c_o, c_p, and c_s is obtained from (16) and (15) (again including the $\binom{K}{b}$ term) as

$$P_1(w_t|c_o, c_p, c_s) = \frac{N_4(w_t|c_o, c_p, c_s)}{\binom{T_s}{c_s}\binom{c_s}{c_o}\binom{W-c_o}{c_p}}. \tag{17}$$

For the case $T_s > W$, our approximation in (13) and (16) does not give accurate results, so we calculate P_1 by the following equation:

$$P_1(w_t|c_o, c_p, c_s) = \frac{N_4(w_t|c_o, c_p, c_s)}{\sum_{w_t=0}^{\min(W, T_s\Theta)} N_4(w_t|c_o, c_p, c_s)}. \tag{18}$$

We can obtain special cases of $P_1(.)$:

1) When $\Theta = 1$: (case $T_s \leq W$)

$$P_1(w_t|c_o, c_p, c_s) = \frac{\binom{W+c_s-T_s-c_o}{w_t+c_s+c_p-T_s}\binom{T_s-c_s}{w_t}}{\binom{W-c_o}{c_p}}. \tag{19}$$

For $T_s \geq W$, no simpler solution can be obtained.

2) When $\Theta = W$: For $c_s = T_s$, $P_1(w_t = 0|c_o, c_p, c_s) = 1$, and 0 otherwise; for $c_s < T_s$, $P_1(w_t = W - c_o - c_p|c_o, c_p, c_s) = 1$, and 0 otherwise.

1) The Joint Distribution of C_o, C_p and C_s: Similarly, we can model C_o, C_p, and C_s as a 3-D Markovian model. We define ρ_s as the load for the total originating calls. Let $\mathcal{L}_s$ denote the set of links connected to s including l_1, and let $\rho_{o,i}$ be the loads of originating calls from s on each link $l_i \in \mathcal{L}_s$. Note that $\rho_s = \sum_{l_i \in \mathcal{L}_s} \rho_{o,i}$. The joint distribution is obtained as

$$P_2(c_o, c_p, c_s) = \frac{\left(\rho_o^{c_o}\rho_p^{c_p}\rho_s^{c_s}\right)/(c_o!c_p!c_s!)}{\Delta} \tag{20}$$

where the normalization factor Δ is obtained by

$$\Delta = \sum_{c_p=0}^{\min\left(W, T_p^{\max}\right)} \sum_{c_o=0}^{\min(W-c_p, T_s)} \sum_{c_s=c_o}^{\min(T_s, D_s W - c_p)} \frac{\rho_o^{c_o}\rho_p^{c_p}\rho_s^{c_s}}{c_o!c_p!c_s!}. \tag{21}$$

In (21), $T_p^{\max}$ is determined by the total number of transmitters of distinct nodes s' originating a route. If the set of such nodes is $\mathcal{N}_s$, then $T_p^{\max} = \sum_{s' \in \mathcal{N}_s} D_{s'} T$.

For SPN model, the Markovian model developed is valid only for the case $T_s \leq W$. When $T_s > W$, this model assumes on the other links of s the number of originating calls to be higher than W for some specific values of c_o. For instance, consider the case when $W = 16$, $T = 10$, $d_s = 2$. We have $T_s = 20$. If $c_o = 2, c_s = 20$, it means that the number of originating calls on the other link of s is $c_s - c_o = 18$, which is not a possible case because there are only 16 wavelengths per link. However, for $T_s > W$, we are still going to use it as an approximate model. An exact multidimensional Markovian model can be developed having two dimensions for originating calls and passing calls per link of a node, but the model would become very complex, and we do not pursue it here.

2) The Joint Distribution of W_t and W_f: This distribution is obtained as

$$P_3(w_t, w_f) = \sum_{c_o=0}^{\min(T_s,W)} \sum_{c_s=c_o}^{T_s} P_1(w_t|c_o, c_p, c_s) P_2(c_o, c_p, c_s). \tag{22}$$

3) End-to-End Blocking Probability: On the receiver side, we define the calls terminating at the destination node d using any of the links connected to d as *total terminating calls*. Let C_d denoted the number of *total terminating calls* on d. Denote by ρ_t, ρ_p, and ρ_d the loads corresponding to terminating calls, passing calls, and total terminating calls, respectively. The total number of transponders of the destination node is $T_d = D_d T$. We need the following distributions.

The Distribution of the Number of Free Receivers: We can calculate the distribution of the number of free receivers R_f on d and C_p on (d, l_h) by utilizing the distribution in (20). P_2 distribution for (d, l_h) also gives the joint distribution of C_t, C_p, and C_d on (d, l_h) with load values of ρ_t, ρ_p and ρ_d respectively. Thus

$$P_5(r_f, c_p|w_f) = \frac{P_2(W - w_f - c_p, c_p, T_d - r_f)}{\sum_{c'_p=0}^{W-w_f} \sum_{c'_s=W-w_f-c'_p}^{\min(T_d, WD_d - c'_p - w_f)} P_2\left(W - c'_p - w_f, c_p, c'_s\right)}.$$

Probability of Nonblocking Given W_t, W_f, R_f, and C_p: We use the distribution in (17) using T_d for the number of transponders with the values of *terminating calls*, *passing calls* on (d, l_h), and *total terminating calls* on d to find the probability of blocking. The corresponding values of c_t and c_d are obtained from w_t, w_f, on l_h and r_f on d. Blocking occurs when there is no receivable wavelength on (d, l_h).

$$P_6(\cdot|w_t, w_f, r_f, c_p) = 1 - P_1(0|c_t, W - c_t - w_t, T_d - r_f)$$

where $c_t = W - w_f - c_p$.

Then, the nonblocking probability for route R is given by

$$B'_R = \sum_{w_f=1}^{W} \sum_{w_t=1}^{w_f} \sum_{r_f=1}^{T_d} \sum_{c_p=\max(0, W-w_f-T_d+r_f)}^{W-w_f} P_3^{(H)}(w_t, w_f) \times P_5(r_f, c_p|w_f) P_6(\cdot|w_t, w_f, r_f, c_p)$$

The blocking probability is given by $B_R = 1 - B'_R$.

H. Model Complexity

In the SPL model, the complexities of the functions N_1, N_2, N_3, N_4, and ζ are $O(TW)$, $O(TW^2)$, $O(W^3)$, $O(TW^5)$, and $O(TW^2)$, respectively. The complexity of P_1 is $O(T^2W)$. The complexity of P_2 for each (s, l_1) pair is $O(TW)$, and for the whole network, it is $O(NDTW)$, where D is the maximum degree of all nodes. P_3, P_5, and B'_R have the complexity of $O(TW^2)$ for a route. The complexity of the path model regardless of SPL and SPN models is $O(HW^5)$ for a route of hop-length H. Therefore, in the SPL model, the overall complexity of calculating blocking probability is $O(|\mathcal{R}| H_m W^5)$, where $|\mathcal{R}|$ is the size of the set $\mathcal{R}$ and H_m is the maximum hop-length of all routes. The complexity is for one iteration, and we use a reduced load approximation in the next section that requires the blocking probability to be computed for a few iterations before it converges.

In the SPN model, the complexities of the functions N_1, N_2, N_3, N_4, N_5, and ζ are $O(DTW^2)$, $O(DTW^2)$, $O(DW^4)$, $O(DTW^7)$, $O(DW^3)$, and $O(DTW^2)$, respectively. The complexity of P_1 is $O(DTW^3)$. P_2 has the complexity $O(DTW^2)$ for a node and $O(NDTW^2)$ for the whole network. P_3, P_5, and B'_R have the complexity of $O(DTW^3)$, $O(DT^2W^3)$, and $O(TW^3)$, respectively, for each route. Therefore, in the SPN model, the overall complexity of calculating blocking is $O(DTW^7)$.

I. Reduced Load Approximation

In all of our calculations of the loads in our model, we have used the offered loads on the routes. However, the actual load on it is only a portion of the offered load since some of the calls are blocked due to nonexistence of a transmittable wavelength end-to-end. When we use the offered load in the analytical model calculations, the model overestimates the blocking probability due to this effect. Especially in the case of high blocking, the calculated blocking values deviate more from the actual ones. This effect can be overcome by using the well-known reduced-load approximation. First, we calculate the initial blocking probability $B_R^{(0)}$ for route R with the offered load value ρ. Then, for every iteration $i = 1, 2, \ldots$, we calculate the new blocking value $B_R^{(i)}$ with the adjusted value of the load $\rho^{(i)} = \rho(1 - B_R^{(i-1)})$. The blocking value converges after a few iterations.

J. Transponder Waveband Assignment Algorithms

Our analytical model assumed random assignment of wavebands to transponders as described in Section III-B. With small number of transponders, we have nodes having transponders with wavebands mostly different from the wavebands at the other nodes as a result of this random strategy. Therefore, transponder blocking becomes the main type of blocking in such cases. For instance, the set of transponder wavebands at a source node s of a route R may have no common wavelength with the set of transponder wavebands at the destination node d. If this happens, then every call on R will be blocked. Therefore, the aim of an assignment strategy should be to increase the probability of transponders at different nodes having wavebands in common. Here, we propose an algorithm for this purpose.

C-Fixed Random Assignment: Let us define $C(0 \leq C \leq T)$ as the number of transponders assigned to the same C distinct wavebands at every node–link pair (n, l_j) in SPL. Similarly for the SPN model, we assign the same $C(0 \leq C \leq TD_{\min})$ distinct wavebands to transponders at every node n. Here $D_{\min} = \min_{n \in 1,2,\ldots,N} D_n$. For $T \leq K$, the remaining $T - C$ or $T_n - C$ (for SPL and SPN, respectively) transponder wavebands are chosen randomly out of $K - C$ wavebands. For $T > K$, the value of C affects the assignment only if $C > \lfloor T/K \rfloor K$. Then, $T - C$ transponder wavebands are chosen randomly out of $K - T \bmod K$ wavebands for the SPL model. The same condition applies for SPN.

Note that if $C = T < K$, then we essentially have a $T \cdot \Theta(< W)$-wavelength system (or $C = T_n < K$ we have $T_n \cdot \Theta(< W)$-wavelength system for SPN).

1) Analytical Model for C-Fixed Assignment: We modify our earlier analytical model in a simple way to find the blocking probability under C-fixed Random assignment. We first compute $P_{3,\text{fix}}^{(H)}(W_t, W_f)$ for the range of C *fixed* wavebands and $P_{3,\text{ran}}^{(H)}(W_t, W_f)$ for the *randomly* assigned wavebands separately by assuming that $\rho_{\text{fix}} = \rho C/T$ and $\rho_{\text{ran}} = \rho - \rho_{\text{fix}}$ are loads for calls within the two sets of wavebands. Also, the number of wavelengths and transponders used in computing the two probabilities are assumed to be $W_{\text{fix}} = C\Theta$ and $T_{\text{fix}} = C$, and $W_{\text{ran}} = W - C\Theta$ and $T_{\text{ran}} = T - C$, respectively.

In this case, the nonblocking probability expressions for the two ranges are exactly the same (but with different values for the loads, number of transponders, and wavelengths, as explained above). The only difference between the two ranges is that the wavebands of the transponders in the fixed range cover the entire band of wavelengths of size $C\Theta$ in the fixed range, whereas they may not in the random range. These two situations are already accounted for in our analysis for the random assignment. Thus

$$B'_{\text{fix}} = \sum_{w_f=1}^{W_{\text{fix}}} \sum_{w_t=1}^{w_f} \sum_{r_f=1}^{T_{\text{fix}}} P_{3,\text{fix}}^{(H)}(w_t, w_f) P_5(r_f|w_f) P_6(w_t, r_f)$$

and

$$B'_{\text{ran}} = \sum_{w_f=1}^{W_{\text{ran}}} \sum_{w_t=1}^{w_f} \sum_{r_f=1}^{T_{\text{ran}}} P_{3,\text{ran}}^{(H)}(w_t, w_f) P_5(r_f|w_f) P_6(w_t, r_f).$$

The blocking probability $B = (1 - B'_{\text{fix}})(1 - B'_{\text{ran}})$.

Special Case $\Theta = 1$: The nonblocking probability calculation for the fixed range becomes simpler since it is guaranteed to have a free receiver for any of the transmittable wavelengths along the route due to the fixed assignment. Thus, the nonblocking probability within this wavelength range is

$$B'_{\text{fix}} = \sum_{w_f=1}^{W} \sum_{w_t=1}^{w_f} P_{3,\text{fix}}^{(H)}(w_t, w_f).$$

For SPN, we assume that the degrees of all nodes are the same, for simplicity of explanation. The loads for the two ranges on a route R depends on the total number of transponders (T_n) at the nodes. They are calculated as $\rho_{\text{fix}} = \rho C/T_n$ and $\rho_{\text{ran}} = \rho - \rho_{\text{fix}}$. Different from SPL, we have $T_{\text{ran}} = T_n - C$. We have the following expressions for blocking

$$B'_{\text{fix}} = \sum_{w_f=1}^{W_{\text{fix}}} \sum_{w_t=1}^{w_f} \sum_{r_f=1}^{T_{\text{fix}}} \sum_{c_p=\max(0, W_{\text{fix}}-w_f-T_{\text{fix}}+r_f)}^{W_{\text{fix}}-w_f} P_{3,\text{fix}}^{(H)}(w_t, w_f) \times P_5(r_f, c_p|w_f) P_6(\cdot|w_t, w_f, r_f, c_p)$$

and

$$B'_{\text{ran}} = \sum_{w_f=1}^{W_{\text{ran}}} \sum_{w_t=1}^{w_f} \sum_{r_f=1}^{T_{\text{ran}}} \sum_{c_p=\max(0, W_{\text{ran}}-w_f-T_{\text{ran}}+r_f)}^{W_{\text{ran}}-w_f} \times P_{3,\text{ran}}^{(H)}(w_t, w_f) P_5(r_f, c_p|w_f) P_6(\cdot|w_t, w_f, r_f, c_p).$$

V. Numerical Results

We apply the blocking analysis for SPL and SPN models and compare it to the actual values obtained from the simulations for two network topologies: 14-node NSFNet topology and a 20-node ring. We assumed a 32-wavelength system ($W = 32$) in our results. We ran simulations using 10^6 call arrivals per trial and 10 trials for each data point. These trials are for averaging out the effect of random waveband assignment used in each trial. The load value we use in our results is the load per route (ρ) as defined in previous sections. All confidence intervals shown are 95% intervals.

We first plot the blocking probability B versus the load ρ for the ring network with $T = 8$ and $\Theta = 4$ in Fig. 8(a) for SPL and SPN models. We have the same comparison in Fig. 8(b) for NSFNet with $T = 12$. We observe a good match between analytical model and simulation results in both topologies, with results being less accurate for the ring. This reduced accuracy in rings is due to the well-known high load correlation between links [15]. Note that our model only includes correlation between adjacent links. The effect of the increasing load is obvious in all curves. We observe that blocking is lower in SPN than SPL, as expected, in both graphs. Sharing of the transponders within a node offers a significant improvement in blocking. We note that the difference between SPL and SPN is higher at lower loads.

We plot B versus Θ in Fig. 9(a) for various values of T for the ring network. For the value of $T = 8$, we don't show the model results in order to be able to distinguish between curves. In all of the curves, there is a significant drop in B until the point where $\Theta = 8$, and no further improvement is achieved for larger values of Θ. Limited tunable transponders with a tuning range value of 8 gives the same performance as the widely tunable transponders in the ring network. We also observe a significant difference between SPL and SPN for $T = 8$ at all values of Θ. However, for $T = 12$, the gap between SPL and SPN has diminished, and, especially for $\Theta \geq 8$, the two models almost give the same results. This is due to the fact that when $T = 12$, there are sufficiently many transponders so that the blocking caused by unavailable transponders (i.e., we call this transponder blocking) is no longer dominant. However, blocking is mainly due to unavailability of wavelengths along the route (i.e., path blocking). Especially with large values of Θ, transponder blocking becomes less and less dominant. We conclude that, for such cases, SPN model does not offer a big advantage over SPL.

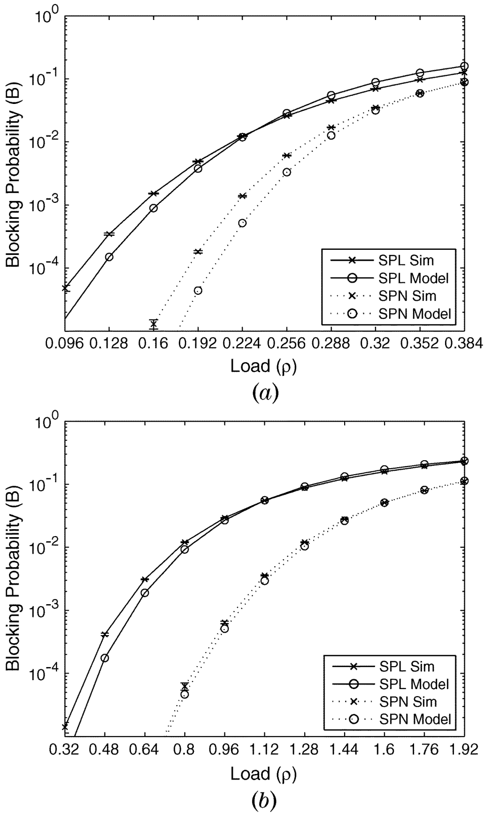

Fig. 8. Blocking probability versus load with $\Theta = 4$ in (a) ring for $T = 8$ and (b) NSFNet for $T = 12$.

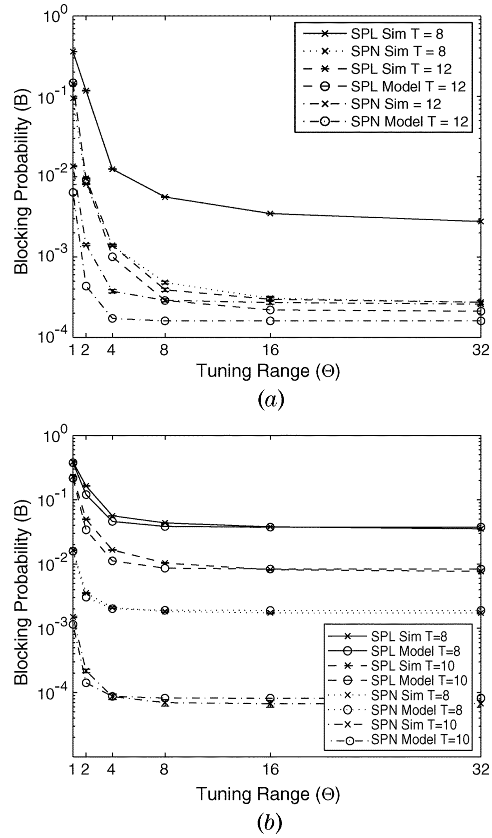

Fig. 9. Blocking probability versus tuning range in (a) ring for $\rho = 0.2240$ and (b) NSFNet for $\rho = 0.64$.

We plot B versus Θ for various values of T for the NSFNet in Fig. 9(b). For all of the curves, a significant drop is observed until the value $\Theta = 4$, and after that, the blocking is almost exactly the same. In this case, there is also a significant amount of difference between SPL and SPN. For these parameters, in the observed load range, transponder blocking is always dominant in NSFNet. Both results for the ring network and NSFNet show that limited tunability is as good as full tunability. Moreover, some tunability in the network gives much better performance than no tunability.

In Fig. 10, we plot B versus T for various values of Θ in the ring network. Fig. 10(a) shows the results for SPL model. The decrease in blocking is more steady for the values of $\Theta = 1$ and $\Theta = 2$. For $\Theta = 2$, the blocking probability settles down after the point $T = 16$. At this point, transponder wavebands cover the entire wavelength range ($T\Theta = 32$), and beyond that point, path blocking becomes more dominant. Similar behavior is observed for $\Theta = 4$, for which the slope of the curve first slows down after the point $T = 8$, where $T\Theta = 32$, and it settles down at the same level after $T = 16$, where $T\Theta = 64$ is a multiple of 32. For the curve of $\Theta = 8$, blocking settles down after $T = 12$, where $T\Theta = 96$. In Fig. 10(b), we show the same set of curves for the SPN model, which has a similar behavior as the SPL model. We observe that the model approximates the blocking less accurately in the SPN case. This is because the SPN model includes one additional variable *total originating calls*. Because of the extra dimension in the Markovian model for this variable, the Markovian model approximation becomes less accurate.

In Fig. 11, we plot B versus T for various values of Θ in NSFNet for both SPL and SPN. In all cases, there is a steady and sharp decrease in blocking as T increases. The number of transponders is very important in determining the blocking performance of the network. Even with large values of T, additional transponders still improve the performance. For both sets of curves ($\Theta = 1$ and $\Theta = 4$), the difference between SPL and SPN becomes larger as T increases. This is again due to transponder blocking being always dominant for NSFNet for the load range considered.

We now analyze the effect of transponder waveband assignment in blocking. We first plot the results of C-fixed random assignment in a ring network for SPL model in Fig. 12. With $T = 6$ in Fig. 12(a), we observe a steady decrease in blocking with increasing C with a load value of $\rho = 0.0256$. Best blocking is achieved with $C = T = 6$ when we have totally fixed assignment of wavebands at every node. The difference between the two end-points is significant (e.g., more than two orders of magnitude). We also observe that our approximate model for the C-fixed assignment is very accurate at the end-points ($C = 0$ and $C = 8$), and the approximation is still reasonable in the midpoints while following the same trend with the actual curves. With a higher load value of $\rho = 0.096$, we no longer have the decrease in blocking with higher C values. On the contrary, there is initially a slight increase in blocking, and with $C = 8$, a slight decrease in the end. The model approximates even the slight changes very accurately. We conclude that in a lightly loaded system such as $\rho = 0.0256$ in our case, to have the

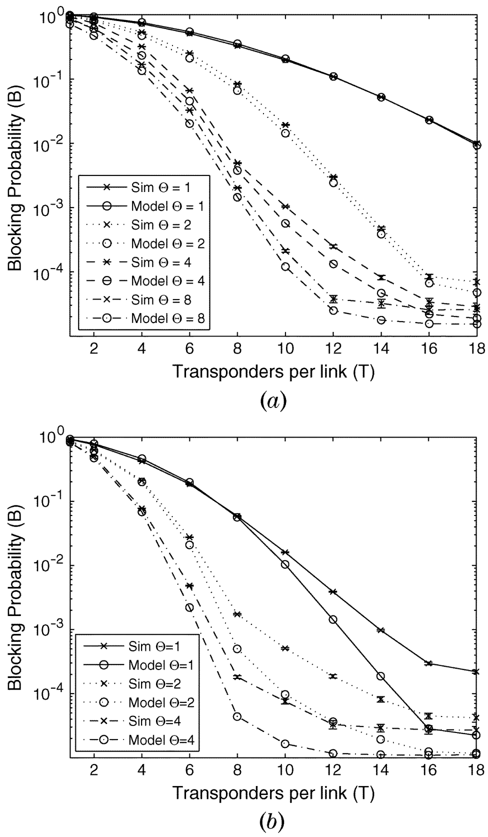

Fig. 10. Blocking probability versus T in ring with load $\rho = 0.192$ (a) for SPL and (b) for SPN.

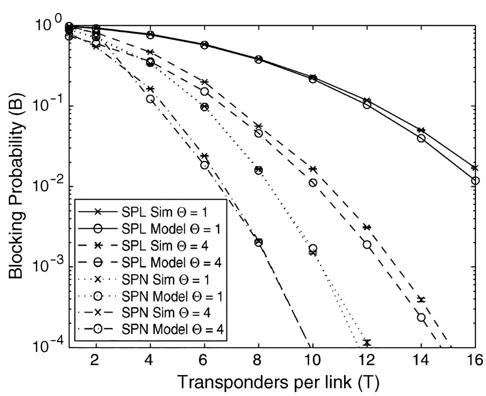

Fig. 11. Blocking probability versus T in NSFNet with load $\rho = 0.64$.

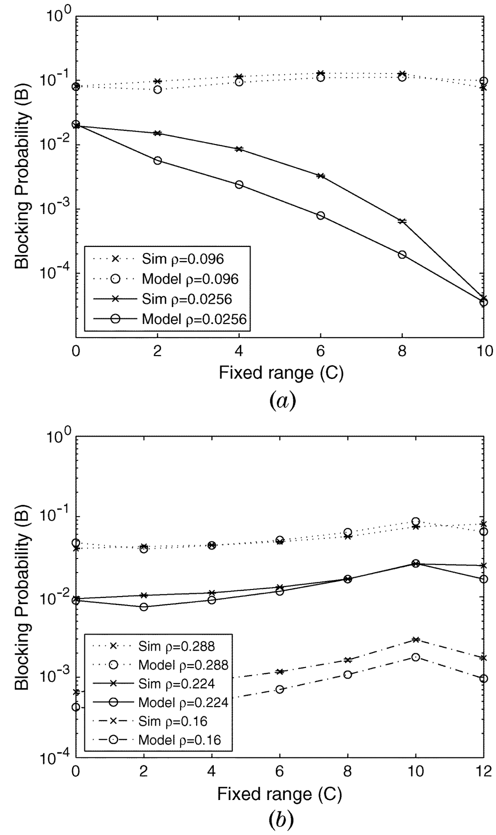

Fig. 12. Blocking probability vs. C in ring for SPL with (a) $T = 10$ and $\Theta = 1$, and (b) $T = 12$ and $\Theta = 2$.

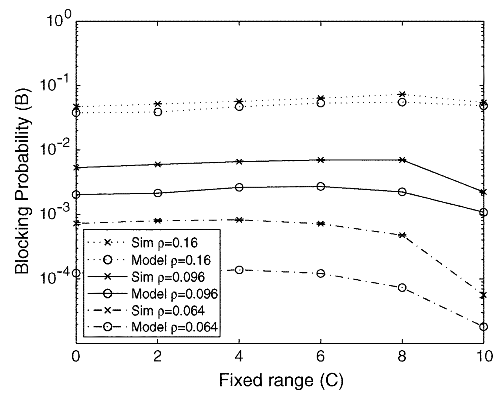

Fig. 13. Blocking probability versus C in ring for SPN with $T = 5$ and $\Theta = 2$.

same wavebands for the transponders at every node gives much better blocking than totally random assignment. However, as we increase the load, blocking starts to dominate due to limited number of wavelengths utilized. Therefore, having fixed wavebands does not offer any advantage. In Fig. 12(b), we show a case with higher T and Θ, $T = 12$ and $\Theta = 2$. The general trend of the curves is a steady increase in C. A little drop in blocking is observed at the point $C = 12$ for two lower load values, but it does not drop below the value of $C = 0$. With large values of T and Θ, transponders are very likely to have the same wavebands at different nodes with random assignment. As a result, transponder blocking is already low, and with higher C values, we increase the load on the fixed wavebands in the network which results in higher path blocking.

In Fig. 13, we show similar results for the SPN model in a ring with $T = 5$, $\Theta = 2$, and various values of load. For $\rho = 0.072$, blocking decreases with larger C, which is not the case for $\rho = 0.16$. The difference between the blocking values of $C = 0$ and $C = 10$ is smaller than in SPL in this case; however the same amount of difference can be obtained with lower values of T in SPN.

VI. Conclusion

In this paper, blocking performance of reconfigurable optical networks is investigated. An introduction on several ROADM architectures and transponder types is given. The use of limited tunable transponders in conjunction with fully reconfigurable ROADMs is evaluated. We defined a constraint on lightpath establishment called *wavelength termination constraint*, which depends on the number of transponders at the nodes and transponders' tunable ranges (wavebands). Based on this constraint, an analytical model is developed for two transponder sharing models, namely share-per-link and share-per-node. Both analytical and simulation results are provided to compare SPL and SPN models and also to see the effects of load, number of transponders, and tuning range on blocking. It is shown that limited tunable transponders are sufficient to achieve the same blocking performance as widely tunable transponders and that limited tunability does provide large improvements over no tunability. SPN model is always advantageous over SPL, but in certain cases, the performance gap between them closes (i.e., dominating path blocking). A heuristic algorithm on transponder waveband assignment is also introduced. The analytical model is modified to analyze the performance of the heuristic. In certain conditions, the blocking is significantly improved by the heuristic, which shows that the waveband assignment has a significant effect on blocking.

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