The Benefits of Wavelength Conversion in WDM Networks with Non-Poisson Traffic

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***Abstract*—The effects of wavelength conversion on wavelength routing optical networks with dynamic non-Poisson traffic are investigated in this letter. A model that characterizes any non-Poisson traffic by its first two moments is utilized. The arrival occupancy distribution of busy wavelengths for this model process is derived and is used to analyze the effects of wavelength conversion. The model predicts that traffic peakedness plays an important role in determining the blocking performance.**

***Index Terms*— Blocking probability, BPP model, moment-matching, wavelength conversion, wavelength routing.**

I. Introduction

THE blocking performance of wavelength routing wavelength-division multiplexing (WDM) networks with dynamic traffic has received considerable attention recently. The performance of such networks is strongly influenced by *wavelength conversion*—the capability of network routing nodes to change the wavelengths of signals. Performance models for networks with and without wavelength conversion under Poisson traffic have been presented in [1], [2]. These models predict that wavelength conversion significantly improves the blocking performance in moderately dense networks such as the mesh-torus, and does not have as significant an effect in sparse and dense topologies such as the ring and the hypercube.

Since Poisson traffic may not be representative of the statistics of input traffic in future optical networks, our objective in this paper is to investigate the sensitivity of those conclusions on the Poisson assumption. We employ a model for non-Poisson traffic called the Bernoulli–Poisson–Pascal (BPP) model, and extend a previous performance model for Poisson traffic [2]. We evaluate the blocking probability and obtain the performance dependence on network parameters such as conversion density and traffic peakedness.

In Section II, we review the BPP framework for modeling non-Poisson traffic. Section III presents the network blocking model. Finally, we use this model to analyze the blocking performance of two networks in Section IV.

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II. Modeling Non-Poisson Traffic

Traffic in circuit-switched networks is often characterized by the busy circuit distribution induced by it in an infinite trunk group [3]. Instead of attempting to study the effect of specific non-Poisson traffic types, we employ a moment-matching technique commonly used in teletraffic theory to study overflow streams in telephone networks. Assuming that the first few moments of the offered traffic are known, the technique consists of choosing an *equivalent process* that yields the same moments and employing it to obtain the busy circuit distribution [4], [5]. In practice, the number of moments that are matched varies from two to four.

We use a widely used model for moment matching called the BPP model. This model has provided fairly accurate results in telephone networks, and is attractive because of its simplicity and the ease with which the model parameters can be obtained from the first two moments. The system is modeled as a birth–death process in which the service times are independent indentically distributed (i.i.d.) exponential, and arrivals occur according to a conditional Poisson process with state-dependent rates.

A. Definitions and Notation

Suppose calls arrive to an infinite-circuit link according to a conditional Poisson process whose rate is λ_k^* in state k. (The state of the system is the number of busy circuits.) Let call holding times be i.i.d. exponential with mean $1/\mu$. Let $\{p_k^*\}$ be the steady-state occupancy distribution, M the expected number of busy circuits (*offered load*), and V the variance of the number of busy circuits. The *peakedness* of the input traffic Z is defined as $Z \overset{\text{def}}{=} V/M$. Peakedness is a measure of a traffic's burstiness and has been used to quantify the deviation of the traffic from Poissonian nature [5]. A traffic is called *smooth*, *regular*, or *peaked* depending on whether Z is less than, equal to, or greater than 1, respectively. Note that a homogeneous Poisson arrival process is regular since it induces a busy-circuit distribution which is Poisson.

Now, consider the link with the number of circuits limited to $F < \infty$. Let the arrival process be a conditional Poisson process with rate λ_k in state k. The *carried load* is the average number of busy circuits, m, when the input traffic is offered to this link. The *unconditional occupancy distribution* is the steady-state distribution $\{p_k\}$ of the number of busy circuits, and the *arrival occupancy distribution* is the steady-state distribution $\{p'_k\}$ of the number of busy circuits observed by an arrival. When the arrival process is non-Poisson, these two distributions are not the same [3].

B. The BPP Model

Given the first two moments (load and peakedness) of an offered traffic, the BPP model consists of an appropriate selection of state-dependent arrival rates $\{\lambda^*_k\}$, depending on whether Z is less than, equal to, or greater than one.

In the Pascal model, the call arrival rate at state k is $\lambda^*_k = \alpha + (k-\alpha)\beta$, $k = 0, 1, \cdots$, for some $\alpha > 0$, $0 < \beta < 1$. The steady-state distribution of the infinite server system is given by the Pascal distribution

$$p^*_k = \left(1 - \frac{\beta}{\mu}\right)^{\gamma} \left(\frac{\beta}{\mu}\right)^{k} \binom{\gamma + k - 1}{k}, \qquad k = 0, 1, \cdots$$

where $\gamma \equiv (\alpha/\beta)(1-\beta)$. Setting $\mu = 1$, one obtains $M = \alpha$, $V = \alpha/(1-\beta)$, and $Z = 1/(1-\beta) > 1$. Given M and $Z > 1$, the arrival process can therefore be modeled as a Markov process with $\mu = 1$, $\alpha = M$, and $\beta = 1 - 1/Z$.

The Bernoulli model is used to model smooth traffic ($Z < 1$). In this model, the state-dependent arrival rates are given by $\lambda^*_k = \lambda(n-k)$, for $k = 0, 1, \cdots, n-1$, and $\lambda^*_k = 0$, for $k \geq n$. The steady-state distribution of the resulting finite-state Markov chain is

$$p^*_k = \binom{n}{k} \left(\frac{\lambda}{\lambda + \mu}\right)^{k} \left(\frac{\mu}{\lambda + \mu}\right)^{n-k}, \qquad k = 0, 1, \cdots, n$$

yielding $M = n\lambda/(\lambda+\mu)$ and $Z = \mu/(\lambda+\mu)$. Given M and $Z < 1$, the arrival process is modeled as a Markov process with $\mu = 1$, $\lambda = (1-Z)/Z$, and $n = M/(1-Z)$. A limitation of the Bernoulli model with a finite link capacity F is that it is applicable only when $Z \geq 1 - M/F$ [4]. The BPP model has been found to produce sufficiently accurate numerical results for blocking probabilities in telephony[1] [4].

The evaluation of blocking probabilities in telephony does not require the arrival occupancy distribution to be computed. As we will see in the next section, this distribution is necessary for the performance analysis of WDM networks without wavelength conversion.

III. Blocking Performance Analysis

This section presents the blocking performance analysis of WDM networks using the BPP model. Calls are assumed to arrive to the network according to a random point process, and each call holds a wavelength on each link of its route for the random duration of the call. The offered traffic for all node-pairs is assumed to be the same, and the offered load per station is denoted by ρ. An arriving call is assigned the shortest path[2] on the physical topology. In each segment between successive wavelength converters along the chosen path, one of the available wavelengths is randomly assigned to the call. The call is blocked if no wavelength is available on a segment. Let the number of wavelengths per fiber be F. To make the analysis tractable, the states of different links are assumed to be statistically independent. We have found this to be a reasonable assumption under Poisson traffic for networks that are not sparse, such as the mesh-torus and hypercube [2].

We first derive the probability distribution of the number of occupied wavelengths on a link as seen by an arriving call request, and then utilize this distribution in the blocking analysis.

A. Arrival Occupancy Distribution

Let λ_i and μ_i denote the call arrival and departure rates when i circuits are busy. The BPP model assumes that the unconditional occupancy distribution in the F-server system is a truncated Pascal, Poisson, or binomial distribution (depending on the value of Z) [4]. To this end, we assume that $\lambda_i = \lambda^*_i, i = 0, 1, \cdots, F-1$.

Let Q_{ij} denote the conditional probability that an arrival in steady state finds the system in state j given that the previous arrival found the system in state i. The arrival occupancy distribution is the solution to the Chapman–Kolmogorov equations $p'_j = \Sigma_{i=0}^{F}\, p'_i Q_{ij}$, $j = 0, 1, \cdots, F$. Since service times are exponentially distributed, Q_{ij} is easily determined [6]. Using the expressions for Q_{ij} and p'_j, it can be shown that $p'_i = (\lambda_i/\mu_i)\, p'_{i-1}$, $i = 1, 2, \cdots F$. Note the difference between the arrival occupancy distribution and the unconditional occupancy distribution which satisfies $p_i = (\lambda_{i-1}/\mu_i)\, p_{i-1}$, $i = 1, 2, \cdots, F$.

The carried load m, which is given by $m = \Sigma_{i=0}^{F}\, ip_i$, is related to the offered load by $m = M(1 - p'_F)$. Hence, it can be shown that $\lambda_F = (\Sigma_{i=0}^{\infty}\, p^*_i \lambda^*_i - \Sigma_{i=0}^{F-1}\, p_i\lambda_i)/p_F$, where λ^*_i is the arrival rate at state i in the infinite-circuit link.

B. Networks without Wavelength Conversion

Given the offered load per station ρ, the offered load per link M can be computed using the uniform traffic and shortest-path routing assumptions. The arrival occupancy distribution on a single link can be computed using this value of M for various values of link peakedness[3] Z using the BPP model. Note that the actual load carried by each link will be less than M due to blocking (which in turn depends on M). We have observed that ignoring this feedback effect does not produce significantly different results when the blocking probabilities are low (on the order of 10^{-2}) while considerably decreasing the computation time.

The probability of blocking on an l-hop path is $t_0^{(l)}$, where $t_n^{(l)}$ is the probability of an arriving call finding n free wavelengths on an l-hop path. $t_n^{(l)}$ is given by the recurrence [2] $t_n^{(l)} = \sum_{i=0}^{F} \sum_{j=0}^{F} R(n|i,j) t_i^{(l-1)} p'_{F-j}$ where p'_i is obtained using the BPP model and the results of Section III-A. Note that $t_n^{(1)} = p'_{F-n}$. From [2], $R(n|i,j) = \binom{i}{n}\binom{F-i}{j-n} \Big/ \binom{F}{j}$ if $\max(0, i+j-F) \leq$

[1] Note that n need not be an integer in the Bernoulli model. Factorials are replaced by Gamma functions when n is not an integer.

[2] In the presence of multiple shortest paths, one path is chosen randomly. Here, the length of a path is the number of physical links or hops on the path.

[3] Link peakedness, the peakedness of a traffic on a link, is a function of the traffic's peakedness at a station [7]. It (as opposed to station peakedness) is used as a traffic parameter in this paper.

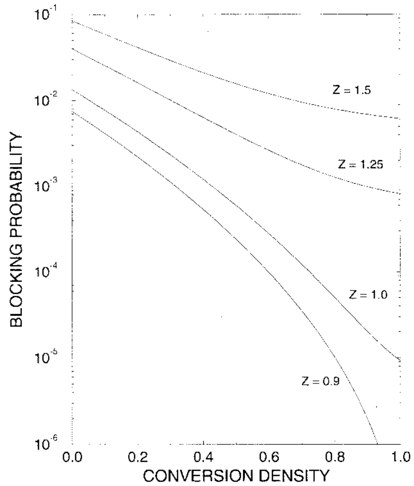

Fig. 1. Blocking probability versus conversion density with offered load of 1.0 per station for a 11 × 11 mesh-torus.

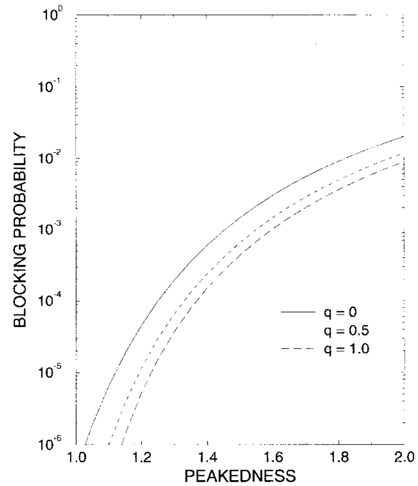

Fig. 2. Blocking probability versus link peakedness for a 128-node hypercube with offered load per station of 1.0.

$n \leq \min(i, j)$, and zero otherwise. The overall blocking performance for networks without wavelength conversion is found by averaging $t_0^{(l)}$ over the hop-length distribution [2]. We will also use $t_0^{(l)}$ to analyze networks with wavelength conversion.

C. Networks with Wavelength Conversion

Following the methodology of [2], we model networks with wavelength conversion by assuming that each node is capable of wavelength conversion with probability q, independently of the other nodes. (The parameter q is called the *conversion density* [2].) The blocking performance we obtain is the ensemble average of the blocking probability over the random placement of a random number of converters with expected value Nq, where N is the number of network nodes. The blocking probability on an l-hop path, $P_b^{(l)}$, can be computed recursively as [2]

$$P_b^{(l)} = t_0^{(l)}(1-q)^{l-1} + \sum_{i=1}^{l-1} \left[1 - (1 - P_b^{(i)})(1 - t_0^{(l-i)})\right] \cdot q(1-q)^{l-i-1}.$$

The blocking performance for different values of traffic peakedness can now be studied. Next, we evaluate the effects of link peakedness and wavelength conversion on call blocking performance for two regular network topologies.

IV. Performance Results

Performance results of applying the analytical model to two network topologies—an 11 × 11 mesh-torus and a 128-node binary hypercube, both with $F = 10$, are presented.

First, we show the blocking probability as a function of the conversion density for the mesh-torus in Fig. 1. It is observed that the blocking probability decreases sharply as conversion density increases for Poisson traffic ($Z = 1$) and smooth traffic ($Z < 1$). However, for peaked traffic, the blocking probabilities do not diminish as rapidly. This suggests that peaked traffic decreases the usefulness of wavelength conversion.

Fig. 2 shows the effect of link peakedness on the blocking probability for the hypercube with a load of $\rho = 1.0$. It is observed that the probability of blocking increases dramatically as the link traffic becomes more peaked. The situation is similar in the mesh-torus (not shown). This is due to the fact that an arrival sees a much more congested system with peaked traffic due to the “bursty” nature of the arrival process.

V. Conclusions

We have developed a methodology for studying the benefits of wavelength converters for non-Poisson dynamic traffic. We employed the BPP model which has been used by teletraffic engineers to characterize the non-Poisson nature of carried and overflow traffic from a trunk. We derived the arrival occupancy distribution for this model and utilized it to analyze the effect of traffic peakedness on wavelength conversion benefits and call blocking performance.

For the two topologies considered, our models predict that for a fixed load, peakedness dramatically increases the blocking probability, and wavelength conversion can only mildly alleviate performance degradation. When the metric of interest is the reduction in blocking probability with wavelength conversion, traffic peakedness reduces the benefits of wavelength conversion.

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